**Optimized Reconstruction of Electron Holograms using the Simplex Algorithmus**

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Off-Axis-Electron-Holography [1] uses a Möllenstedt biprism [2] in the electron beam to interfere the object wave with the reference wave. Quantum mechanics describes the electron wave at the position $\vec{\ }$ and with the de Broglie wave vector $\vec{\ }$ as $\Psi = \quad p \vec{\ }\vec{\ }$. The image wave $\Psi \ \vec{\ } = \ \vec{\ } \quad - \ p\vec{\ }\vec{\ } + \Phi \ \vec{\ }$ is modulated in amplitude $A$ and phase $\Phi$ by the complex object transparency. In the image plane, the intensity $\vec{\ }$ of the electron beam results from the interference of image wave $\Psi$ and reference wave $\Psi$:

$$= \left|\Psi + \Psi\right| \ ,$$

yielding

$$\vec{\ } = \ + \quad + \quad \cdot \qquad p\vec{\ }\vec{\ } + \Phi$$

with $\vec{\ } = \vec{\ } - \vec{\ }$ and contrast $V$, depending on the coherence of the source and mechanical and electrical imperfections; we do not consider the image transfer function, because it does not affect further discussion. In the case of finite numbers of detected electrons, noise has to be added by $\vec{\ }$, yielding the measured intensity

$$\vec{\ } = \ + \quad + \quad \cdot \qquad p\vec{\ }\vec{\ } + \Phi \ + \quad \vec{\ }$$

Our goal is to identify the amplitude $A$ and the phase $\Phi$ of the image wave. Therefore, a system of intensity equations can be solved. In the experimental situation, we measure the intensities $\quad = \quad \vec{\ }$ at the CCD pixel positions $\vec{\ }$ (fig. 1). If the image wave is constant over the input area of $n$ pixels, we can collect the different intensities to an intensity vector $\vec{\ } = \vec{\ } \quad \vec{\ }$. This vector describes a position in the $n$ dimensional input data space , which should point to a two dimensional surface $\quad \Phi \ \in$ , with the parameters amplitude and phase. Due to the noise $\vec{\ }$, however, the intensity vector $\vec{\ }$ misses the surface $\quad \Phi$. The values and $\Phi$ corresponding to surface point $\vec{\ }$ which has the smallest distance to $\vec{\ }$ are the best available approximation for $A$ and $\Phi$. In contrast to classical reconstruction, the reconstructed image wave is a nonlinear function of the intensities. This yields more information because the whole hologram and not only the sideband is used.

There are different approaches to find the point $\vec{}$ on the surface. One approach uses neural nets [3,4], we use in this case the simplex algorithm. The downhill simplex method is due to Nelder and Maed [5]. It starts with a simplex in the parameter space and then approximates numerically the point on the surface which satisfies the condition

$$\left\| \vec{} - \vec{} \quad \Phi \right\| < \left\| \vec{} - \vec{} \quad \Phi \right\| \forall \vec{} \subset \quad \Phi \, .$$

The search ends when the decrease of the distance to the surface between consecutive iterations becomes smaller than a given $e$. For reconstruction, the input area is moved over the whole hologram and the values and $\Phi$ are calculated and written to an array.

In simulated holograms subject to poisson and thermal noise, this process of reconstruction shows the best results. The disadvantage is the immense amount of numerical processing, because, for the determination of one pair , $\Phi$ , the

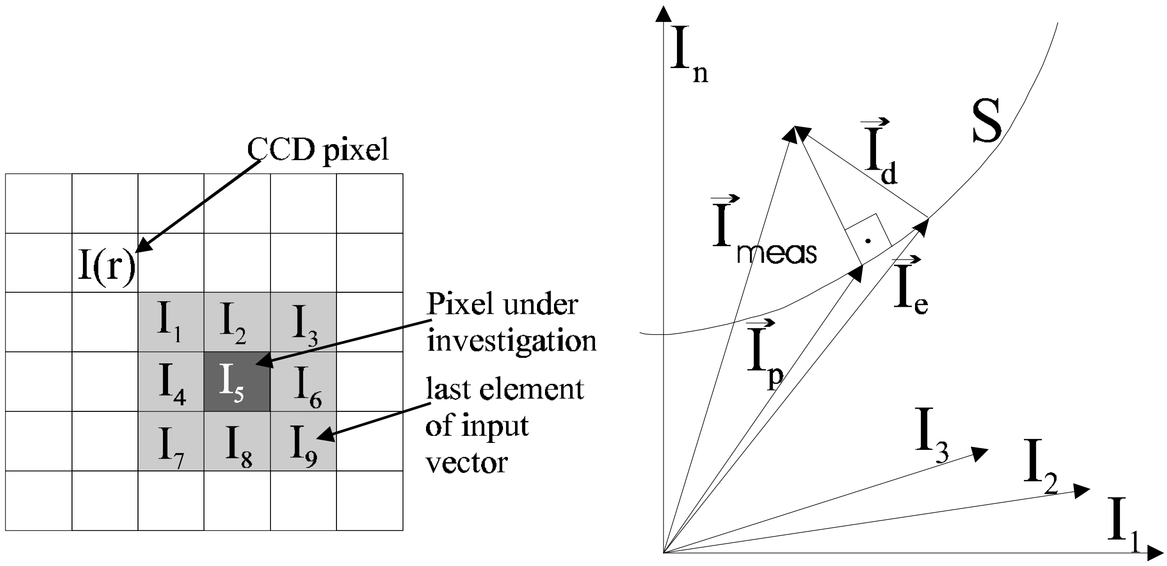


Fig. 1: Detail of the CCD array. The pixel under investigation measures the intensity $I_5$. The simplex uses the intensity information $I_1$ ... $I_9$ to find amplitude and phase of pixel 5 in the hologram

simplex algorithm takes about 100 iterations, and for each iteration, the intensity equation has to be calculated times.

This gets even worse if the assumption that the image wave is constant over the input area no longer holds. In this case, additional parameters have to be introduced, e.g. the linear and quadratic taylor expansion coefficients of the image wave. With increasing number of parameters the efficiency and reliability of the simplex algorithm rapidly decreases. However, one can take advantage by easy parallel implementation of this reconstruction process from further development in computer architecture.

REFERENCES
1. Lichte H., (1986) Ultramicroscopy **20**, 293-304
2. Möllenstedt G., (1957) Düker H. Zeitschrift für Physik, Vol. **145** 377 - 397
3. Heindl E. et.al. (1996) Ultramicroscopy, in press
4. Meyer R. Heindl E. (1996) Poster EUREM ´96 submitted
5. Press W. H. (1986) Numerical Recepies, Cambridge University Press, p. 289

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Fig. 2: The possible intensities without noise lie on the surface S. The point is the surface point closest to the measured value , which is displaced by the noise vector .